Dear Editor,

Thank you for giving us the opportunity to submit a revised draft of the manuscript “Numerical integration scheme for coupled elastoplastic-viscoplastic constitutive law for tunnels” for publication in the International Journal of Geomechanics. We appreciate the time and effort that you and the reviewers dedicated to providing feedback on our manuscript and are grateful for the insightful comments on and valuable improvements to our paper. We have incorporated most of the suggestions made by the reviewers. Please see below, in blue, for a point-by-point response to the reviewers’ comments and concerns. All page numbers refer to the revised manuscript file.

**REPLIES TO EDITOR’S E-MAIL**

• **Please note the byline on your manuscript text doesn’t match the manuscript data entered (add/edit/delete authors step). Please correct this and resubmit.**

The author´s name was changed in the Editorial Management system from Felipe Quevedo to Felipe Pinto da Motta Quevedo to match the manuscript data.

• **Please upload your revised manuscript file in Microsoft Word or LaTex format. If you using LaTex, you may submit a PDF file for review. Please see our LaTex instructions on the Author main page for more information.**

The revised manuscript file is submitted in pdf format together with the project Latex files.

• **Remove the figures from your manuscript text and upload them separately (one figure per file) in TIFF, EPS or PDF format. Also, please make sure to reference the figure number in each file name.**

The figures were removed from the manuscript text and uploaded as separate files.

• **Double-spaced list of figure captions. Please provide a double-spaced list of figure captions with your submission. This can be at the end of your manuscript text or uploaded as a separate Word file. Also, please make sure if you have figures labeled as Figure 1a, 1b, etc. that the captions for these parts of the figure are included in your Figure Caption List.**

The list of figure captions is now provided at the end of the revised manuscript.

• **Embedded Tables. Please remove tables from within the text of your paper and place them at the end of your manuscript after the references . If you upload them separately, please make sure they are uploaded in Microsoft Word/LaTex format.**

Tables are at provided the end of the revised manuscript after the references.

**Also, please note in order to clarify math for copyeditors, please ensure that you use boldface for matrices, vectors, and tensors; italics for all variables and lowercase Greek letters; and roman for all numerals, uppercase Greek characters, and mathematical operators.**

The manuscript has been carefully checked to clarify math for copyeditors, meeting the above recommendations.

**Please submit the revised manuscript and a detailed response to the reviewers' criticisms by logging onto the Editorial Management system at https://www.editorialmanager.com/jrngmeng/ and clicking on the "Submissions Needing Revision" link.**

The revised version of the manuscript has been submitted onto the Editorial Management system together with a detailed response to the reviewers' comments.

**RESPONSES TO REVIEWER 1**

**This is very interesting research work to apply the hyperelasticity to the geo-material. In addition, this is the suitable method to employ this constitutive law to the integral of the implicit method. However, it is difficult to understand it. The authors should friendly revise some explanation in each model's part.**

**1. The reviewer does not think that it is necessary to explain according to Hyperelasticity. However, the authors should explain it more concisely so that the reader can understand it. Why did the authors employ Hyperelasticity to the constitutive model of the geo-material. The authors should explain the reason in detail.**

The constitutive model adopted in the paper to describe the behavior of the geo-material is not formulated within the context of hyperelasticity. It actually consists of a coupled elastoplastic-viscoplastic model , which is formulated in the framework of infinitesimal strains. For the sake of clarity, the equations that express the specific free energy were therefore removed from the manuscript text.

**2. The potential function of hyperelasticity, ψe, should be described.**

As stated in the above comment, the constitutive model of the geo-material does not refer to hyperelasticity, but to an infinitesimal coupled elastoplastic-viscoplastic model. For the sake of clarity, the equations that express the specific free energy were therefore removed from the manuscript text.

**3. The authors should declare that the cohesion, c, is a variable when it is employed at the yield function.**

Following the reviewer’s recommendation, the cohesion c is declared as a variable when it is introduced to define the yield function.

**4. The authors should show the concrete form of the plastic potential function. In addition, they should explain why they use such a function.**

Referring to the Reviewer’s comment regarding the plastic potential function, the last paragraph of Sub-Section “Plastic flow rule” has been reformulated (lines 103 to 108) :

“[…] In the particular case of Drucker-Prager potential flow,  corresponding to , , . From the numerical viewpoint, the main advantage of using such a potential function lies in the fact it is a smooth function. Another advantage is that it can simulate the volume variation during the evolution of plastic deformations (dilation). This effect is commonly introduced through non-associated plasticity, adopting, instead of the friction angle a dilatancy angle in the potential function .”

**5. There are many variables in the hardening rule, and the authors should explain them such as the magnitude relationship of cp, ci, cr, etc., in detail. In addition, the authors should also explain the difference between zones. It should also show a comparison with the response of the actual material.**

The hardening rule described by the variations of the material cohesion with the state variable (equivalent plastic deformation) is now better described by introducing new Figure 1. The latter illustrates the variations of the piecewise linear function , as well as the different zones I to IV. Figure 1 also provides a comparison between the model hardening law and triaxial rock tests.

**6. The reviewer thinks it is better to explain the validity of the constitutive rule by showing some analysis examples (stress-strain relations) such as triaxial tests. At that time, it is better to compare it with the actual experimental data.**

As explained in the response to the preceding Reviewer’s comment, new Figure 1 shows a comparison of the model predictions with experimental data from triaxial tests on the Boom Clay Rock Mass (Rousset, 1988).

**7. Equations (2), (3), (4): The parameter, q, in the proposed model may be variable. In essence, it is a model that the cohesion, c, changes due to plastic deformation. However, it is difficult to understand. The authors should describe and explain the yield function and the cohesion in the yield function.**

The yield function and related parameters are better defined in the revised draft (see comment after Equation (4)).

**8. Equation (5): The reviewer thinks that it is better to remove equation (5) since it is difficult to understand it.**

Following the Reviewer’s suggestion, Equation (5) of the original submission has been removed from the manuscript text.

**9. Equation 6: The authors indicated the Load angle, θ. However, it was not employed in the yield function. On the other hand, it was employed in the plastic potential. The authors should explain it.**

In a first step, expression (3) (Equation 6 in the original submission) describing the flow function of isotropic materials is written in the general form as a function of all the stress invariants. In the sequence,  that stands for the effects of third stress invariant  does not appear because the Drucker-Prager surface used for the numerical simulations is independent of this stress invariant. At that respect, the following sentence has been introduced in line 89 after Equation (5):

“The Lode angle does not appear because the Drucker-Prager surface does not depend on this angle in the deviator plane”.

The potential function is also described in its general form, however, the Drucker-Prager potential function used in the analyses does not depend on this term: (as stated in line 104).

**10. In the geomechanics, the compression is often implicitly positive. At the first appearance, σ, the authors may declare the tension is positive.**

Convention of positive stress in tension is now clearly stated in the revised draft. The following sentence has been added after Equation (2) – line 66:

“[…] Sign convention of positive stress in tension is adopted throughout the paper.”

**11. Equation 10: The formula of the function, g, should be described.**

As indicated in the response to comment 4, function g adopted in the analyses is now better described in the revised draft.

**12. Equation 11: g3 should be solved and the formula of dJ3/dσ should be also described.**

Expression  is now provided in Equation (7) of the revised manuscript.

**13. Equation 24: The first term of the left hand side is the strain rate. Therefore, "dot" is necessary.**

The first term of the left hand side in Equation (20) (Equation 24 in the original submission) has been corrected.

**14. Equation 26: It is normally use to ε = εe+εvp. It is rear to separate between plastic strain, εp, and εvp. The authors should explain how εp and εvp were calculated individually.**

From a physical viewpoint, the model shall account for both instantaneous irreversible strains (plasticity) and delayed irreversible strains (viscoplasticity). In this context, additive form for the total strain rate is considered in the modeling. The latter consists in three elementary rheological models associated in series. The individual contribution of each component is described separately.

The first paragraph of Section “ELASTOPLASTIC-VISCOPLASTIC CONSTITUTIVE MODEL” (line 180) has been reformulated as:

“The proposed elastoplastic-viscoplastic model is formulated from the serial association of the constitutive models described in the preceding sections, i.e., , which leads to the following constitutive relationship:



This association can be seen in the one-dimensional representation of Fig. 2. […]”

**RESPONSES TO REVIEWER 2**

**The concepts presented in the paper are available in standard texts. May focus on the work done by the Authors using UPF.**

**Elastoplastic-viscoplastic constitutive formulations are presented in the paper. Generally, such coupled analysis is required if rate-independent (instantaneous) analysis needs to be augmented with time-dependent yielding analysis such as the effect of creep. The concept presented in the paper is well documented in standard literature (both rate-independent and time-dependent). Coupling of these two methods are also presented in several literatures as also mentioned by the authors. In that sense, the paper does not provide any new information.**

From a physical viewpoint, the model shall account for both instantaneous irreversible strains (plasticity) and delayed irreversible strains (viscoplasticity). In this context, additive form for the total strain rate is generally considered in coupled analyses. Although each separate component of the geo-material deformation is classically well documented in standard literature, few models specifically devised for dealing with coupled plasticity-viscoplasticity are available and they address very specific situations regarding the plastic and viscoplastic flow surfaces. A flowchart for a general scheme, such as that presented in Figure 4, is rarely presented for these coupled models.

**However, a coupling integration algorithm is presented in UPF platform of ANSYS which may have some interest to the readers of this journal. It is encouraged that the authors focus their paper in that direction rather than elaborating the concept already available in the standard books and literature. While revising the paper, authors may highlight the following:**

**1. Dilation of rock materials is neglected in the present study. Dep in elasto-plastic analysis will be unsymmetrical in non-associative flow. Please make comments on symmetrisation techniques or comment whether one needs a non-symmetrical solver. Authors may refer to the following symmetrisation papers.**

**Pande et al., 1986, “Symmetric tangential stiffness formulation for non-associative plasticity”, Compu Geotech, 2(2) 89-99**

**Deb et al., 2013, “Generalized symmetric formulation of tangential stiffness for nonassciative plasticity”, J. of Engg. Mech, Vol 139, issue 2.**

The effects of dilation of rock materials are accounted for in the constitutive model as well in the numerical algorithm. As a matter of fact, the plastic and viscoplastic models (Drucker-Prager) involve irreversible volume variation (dilation) material deformation.

Regarding the non-symmetric solver, the following paragraph has been added after Equation (31) – line 257:

“In non-associated plasticity, the constitutive modulus matrix is not symmetric and its update leads to a non-symmetric global stiffness matrix, thus requiring a non-symmetric solver for the global equilibrium iterations. Symmetrization techniques of the constitutive stiffness matrix, such that proposed in Pande and Pietruszczak. (1986) or in Deb et al. (2013), can be used. It is observed that the algorithm converges even not updating the constitutive modulus. Although it is optional, the calculations performed in this paper make use of this update.”

**2. Express momentum balance equation in static condition and make comments on the increment of external load, especially whether it will be time dependent or not. Generally, for elasto-plastic analysis delta(t) is a pseudo-parameter, however, it is an important parameter for visco-plasticity. Will the stress corrections in elasto-plastic analysis now be depended on delta(t)?**

Referring to the momentum balance equation, the following text has been added at the beginning of Section “VALIDATION OF THE MODEL” – Line 318:

“Before proceeding to the model validation, it is emphasized that the momentum balance equations in quasi-static conditions read in incremental form , where  and  are respectively the stress increment and the increment of external load. In the case of tunnel simulation considered in the sequel, the increment of external load is time dependent since it is associated with the process of ground excavation and tunnel advancement (rock elements are removed).”

Regarding the time dependence of the mechanical fields in the coupled elastoplastic-viscoplastic analysis, the following paragraph has been added at the end of Sub-section “Integration of elastoplastic-viscoplastic constitutive equations” - line 312:

“In elastoplastic analyses, time is a kinematical parameter that marks the load history and control the incremental solution, but it does not influence the constitutive relationships. In coupled elastoplastic-viscoplastic analyses, the elastoplastic component of constitutive relationships becomes time-dependent since it explicitly depends on the viscoplasticity behavior. In particular, the stress corrections in the elastoplastic step of the solution procedure shall a priori depend on  ”

**3. Authors have assumed that increment of strain(vp) will be estimated first and stress will be updated before elasto-plastic analysis starts. One would think it may happen in the reverse way.**

From a physical viewpoint, the plastic strain increment refers to instantaneous component of deformation increment whereas the viscoplastic strain increment refers to the delayed one. However, the procedure for numerical solution evaluates first the viscoplastic strain increment and then implements in a subsequent analysis the elastoplastic integration algorithm. For the sake of clarity, the first paragraph of Sub-section “Integration of elastoplastic-viscoplastic constitutive equations” – line 303 has been reformulated as:

“As viscoplasticity strains are integrated using a semi-implicit rule in which all variables are calculated with known stresses (from substep ), the viscoplastic strain increment is computed first. Subsequently, it is deducted from the total strain increment evaluated by elastic predictor in the elastoplasticity algorithm. This is actually the correct sequence for computation modeling of the coupling, since elastoplasticity algorithm updates the stress state for substep  in order to verify the criterion . If the plastic strain were calculated before the viscoplastic strain, the criterion value would be . […]”

**4. Define ci, cp, cr in equation 13.**

The hardening rule described by the variations of the material cohesion with the state variable (equivalent plastic deformation) is now better described by introducing new Figure 1. The latter illustrates the variations of the piecewise linear function , as well as the different zones I to IV. Figure 1 also provides a comparison between the model hardening law and triaxial rock tests.

**5. Elaborate on the UPF code in ANSYS for the benefit of the readers. This is probably the novelty of this paper. Authors may schematically present the code block in Fortran. Title of the paper may be changed accordingly.**

The procedure for computation implementation is described by the flowchart in Figure 4. The idea of presenting the flowchart instead of focusing on the Fortran code implementation is that the model can be implemented in any software and any programming language. In this way, it will not be restricted to ANSYS users and Fortran 77 readers. However, reference to Quevedo PhD thesis (2021), which provides the Fortran code and related implementation details was added at line 310 of the revised draft:

“[…] The implementation in the USERMAT subroutine in FORTRAN77 can be found in Quevedo (2021)”

**6. Is superscript p valid in equation given in line 214?**

Subscript p has been introduced in the Equation of line 218 (line 214 in the original submission).

**7. The example problem is solved considering associative flow. Analyze the same example considering non-associative flow rule.**

Following the Reviewer’s suggestion, additional simulations have been performed considering non-associative flow rule. The results of this analysis are shown in Figure 8.

A comment regarding this aspect has been added at the end of Sub-section “PARAMETRIC ANALYSIS, RESULTS AND DISCUSSION”

**8. Line 318: ratio not ration.**

The term has been corrected (in line 339).

**The paper needs major revision as mentioned above.**

**RESPONSES TO REVIEWER 3**

**1. Authors should review the consistency and notation of all equations.**

The manuscript has been carefully checked and corrected, the consistency and notation have been reviewed.

**2. The "zones" in equations 13 and 15 are not well explained (Authors could make a graphic showing these different zones)**

The hardening rule described by the variations of the material cohesion with the state variable (equivalent plastic deformation) is now better described by introducing new Figure 1. The latter illustrates the variations of the piecewise linear function , as well as the different zones I to IV. Figure 1 also provides a comparison between the model hardening law and triaxial rock tests.

**3. The literature review should be reduced and more details should be given on the coupling of the constitutive models**

The coupled elastoplastic-viscoplastic model is the junction of two components (elastoplastic and viscoplastic). For the sake of clarity, it is therefore essential to first describe the main features of both the plastic and viscoplastic models involved in the formulation of the coupled one. In that respect, several additional comments referring to the formulation of coupled model and related numerical aspects have been added in the revised drafts.

A) Last paragraph of Section “ELASTOPLASTIC-VISCOPLASTIC CONSTITUTIVE MODEL” (line 188):

“An interesting aspect of the coupled model, using the Drucker-Prager criteria  and for plasticity and viscoplasticity with , is that the evolution of local mechanical fields are entirely controlled by the cohesion. In particular, when  and  the solution is purely elastic. Besides, the purely elastoviscoplastic solution is retrieved when , whereas purely elastoplastic solution is obtained when .”

B) First paragraph of Sub-section “Integration of elastoplastic-viscoplastic constitutive equations” – line 303 has been reformulated as:

“As viscoplasticity strains are integrated using a semi-implicit rule in which all variables are calculated with known stresses (from substep ), the viscoplastic strain increment is computed first. Subsequently, it is deducted from the total strain increment evaluated by elastic predictor in the elastoplasticity algorithm. This is actually the correct sequence for computation modeling of the coupling, since elastoplasticity algorithm updates the stress state for substep  in order to verify the criterion . If the plastic strain were calculated before the viscoplastic strain, the criterion value would be . […]”

C) The following paragraph has been added at the end of Sub-section “Integration of elastoplastic-viscoplastic constitutive equations” - line 312:

“In elastoplastic analyses, time is a kinematical parameter that marks the load history and control the incremental solution, but it does not influence the constitutive relationships. In coupled elastoplastic-viscoplastic analyses, the elastoplastic component of constitutive relationships becomes time-dependent since it explicitly depends on the viscoplasticity behavior. In particular, the stress corrections in the elastoplastic step of the solution procedure shall a priori depend on  ”

**4- The ANSYS APDL script for the FEM model and the USERMAT subroutine in FOTRAN for the rock constituent model should be publicly available datasets. It is recommended to use for example "Datasets related to this article can be found at [INSERT PERMANENT URL(s) TO BE LINKED TO DATASET], hosted at [NAME OF HOSTING REPOSITORY] ([CITATION TO DATASET])".**

The procedure for computation implementation is described by the flowchart in Figure 4. The idea of presenting the flowchart instead of focusing on the Fortran code implementation is that the model can be implemented in any software and any programming language. In this way, it will not be restricted to ANSYS users and Fortran 77 readers. However, reference to Quevedo PhD thesis (2021), which provides the Fortran code and related implementation details was added at line 310 of the revised draft:

“[…] The implementation in the USERMAT subroutine in FORTRAN77 can be found in Quevedo (2021)”

and the following sentence has been added at line 334:

“[…] The FEM model in ANSYS APDL script can be found in Quevedo (2021).”

It is recalled the following sentence regarding the following data availability statement is provided at line 378 of the original submission:

“Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request. (ANSYS APDL script for FEM model and USERMAT subroutine in FORTRAN 77 for constitutive rock mass model).”